$$\int_{D\nearrow green} Pdx + Qdy = \int_0^1 \int_{-y}^0 \left(2xye^{x^2} - 2y\right) dxdy + \int_1^2 \int_{-\sqrt{1-(y-1)^2}}^{\sqrt{1-(y-1)^2}} (2xye^{x^2} - 2y)\, dxdy$$

Integramos

$$\int_0^1 2y\left(\frac{e^{x^2}}{2} - x\right)_{-y}^{0} dy + \int_1^2 2y\left(\frac{e^{x^2}}{2} - x\right)_{-\sqrt{1-(y-1)^2}}^{\sqrt{1-(y-1)^2}} dy$$

Evaluando límites

$$\int_0^1 2y\left(\frac{1}{2} - \frac{e^{y^2}}{2} - y\right) dy + \int_1^2 2y\left(\frac{e^{1-(y-1)^2}}{2} - \sqrt{1-(y-1)^2} - \left(\frac{e^{1-(y-1)^2}}{2} + \sqrt{1-(y-1)^2}\right)\right) dy$$

Simplificando

$$\int_{D\nearrow} Pdx + Qdy = \underbrace{\int_0^1 y - ye^{y^2} - 2y^2 dy}_{I_1} + \underbrace{\int_1^2 -4y\sqrt{1-(y-1)^2} dy}_{I_2}$$

Resolviendo la primera integral

$$I_1 = \int_0^1 y - ye^{y^2} - 2y^2 dy = \left(\frac{y^2}{2} - \frac{e^{y^2}}{2} - \frac{2}{3}y^3\right)_0^1 \quad => \quad I_1 = \left(\frac{1}{2} - \frac{e}{2} - \frac{2}{3}\right) - \left(-\frac{1}{2}\right) \quad => \boldsymbol{I_1 = \frac{1}{3} - \frac{e}{2}}$$

Resolviendo la segunda integral

$$I_2 = \int_1^2 -4y\sqrt{1-(y-1)^2} dy \quad CV \quad a = y - 1 \quad => \quad da = dy \ : \begin{cases} y = 1 \ => \ a = 0 \\ y = 2 \ => \ a = 1 \end{cases}$$

Queda

$$I_2 = \int_0^1 -4(a+1)\sqrt{1-a^2} da$$

Cambio trigonométrico $a = \sin(\phi) \quad => \quad da = \cos(\phi)\, d\phi \quad : \begin{cases} a = 1 \ => \ \phi = \frac{\pi}{2} \\ a = 0 \ => \ \phi = 0 \end{cases}$

Entonces

$$I_2 = \int_0^{\frac{\pi}{2}} -4(\sin(\phi) + 1)\sqrt{1 - \sin^2(\phi)} \cos(\phi)\, d\phi$$

$$I_2 = \int_0^{\frac{\pi}{2}} -4(\sin(\phi) + 1)\cos^2(\phi)\, d\phi \quad => \quad I_2 = -4\int_0^{\frac{\pi}{2}} (\sin(\phi)\cos^2(\phi) + \cos^2(\phi)) d\phi$$

$$I_2 = -4\left(\underbrace{\int_0^{\frac{\pi}{2}} (\sin(\phi)\cos^2(\phi))d\phi}_{I_3} + \underbrace{\int_0^{\frac{\pi}{2}} (\cos^2(\phi))d\phi}_{I_4}\right)$$

Para la integral 3 se tiene que hacer una sustitución

$$u = \cos(\phi) \quad => \quad du = -\sin(\phi)\, d\phi \quad : \begin{cases} \phi = 0 \quad => \quad u = 1 \\ \phi = \frac{\pi}{2} \quad => \quad u = 0 \end{cases}$$

Por lo que

$$I_3 = -\int_1^0 u^2 du \quad => \quad I_3 = -\left(\frac{u^3}{3}\right)_1^0 \quad => \quad I_3 = \left(\frac{1}{3}\right)$$

Para la integral 4 debemos recordar la identidad del ángulo medio

$$I_4 = \int_0^{\frac{\pi}{2}} \frac{(1+\cos(2\phi))}{2} d\phi \quad => \quad I_4 = \frac{1}{2}\left(\phi + \frac{\sin(2\phi)}{2}\right)_0^{\frac{\pi}{2}} \quad => \quad I_4 = \frac{1}{2}\left(\frac{\pi}{2} + \frac{\sin(\pi)}{2}\right) \quad => \quad I_4 = \frac{\pi}{4}$$

Para concluir

$$\int_{D\nearrow green} Pdx + Qdy = \mathbf{\frac{1}{3}} - \boldsymbol{\frac{e}{2}} - \mathbf{4}\left(\mathbf{\frac{1}{3}} + \boldsymbol{\frac{\pi}{4}}\right) \quad => \quad \int_{D\nearrow green} Pdx + Qdy = -1 - \underbrace{\boldsymbol{\pi}}_{\boldsymbol{MisterioResuelto}} - \frac{e}{2}$$